

$$\textcircled{3} \vec{F}(x, y, z) = yz\vec{i} + xz\vec{j} + xy\vec{k}$$

$$\vec{r} = t\vec{i} + t^2\vec{j} + t^3\vec{k} ; 0 \leq t \leq 1$$

$$d\vec{r} = \vec{i} + 2t\vec{j} + 3t^2\vec{k} \quad \vec{F}(\vec{r}(t)) = t^5\vec{i} + t^4\vec{j} + t^3\vec{k}$$

$$W = \oint_C \vec{F} \cdot d\vec{r} = \oint_C (\vec{F}(\vec{r}(t)) \cdot \frac{d\vec{r}}{dt}) dt$$

$$= \int_0^1 (t^5\vec{i} + t^4\vec{j} + t^3\vec{k}) (\vec{i} + 2t\vec{j} + 3t^2\vec{k}) dt$$

$$= \int_0^1 (t^5 + 2t^6 + 3t^5) dt = \int_0^1 6t^5 dt$$

$$= t^6$$

$$\textcircled{4} \oint (3x - 4y) dx + (4x + 2y) dy$$

$$C = \frac{x^2}{16} + \frac{y^2}{9} = 1$$

$$0 \leq t \leq 2\pi$$

parametric equation

$$x = 4 \cos t \quad y = 3 \sin t$$

$$dx = -4 \sin t dt \quad dy = 3 \cos t dt$$

$$\int_0^{2\pi} (12 \cos t - 12 \sin t) - 4 \sin t dt + (16 \cos t + 6 \sin t) 3 \cos t dt$$

$$\int_0^{2\pi} -48 \cos t \sin t dt + 48 \sin^2 t dt + 48 \cos^2 t dt + 18 \sin t \cos t dt$$

$$\int_0^{2\pi} -30 \cos t \sin t dt + 48$$

$$= -15 \sin^2 t + 48t = 48 \times 2\pi = 96\pi$$

$$\boxed{5} \quad I = \int_{(1,2)}^{(3,4)} (6xy^2 - y^2) dx + (6x^2y - 3xy^2) dy$$

Sol.

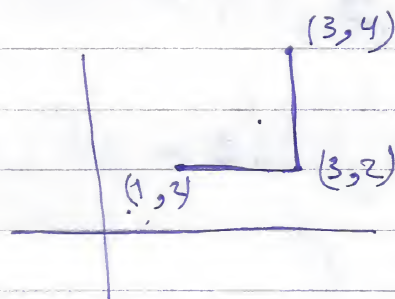
$$\text{Let } (6xy^2 - y^2) = M$$

$$\frac{\partial M}{\partial y} = (12xy - 2y)$$

$$\text{Let } (6x^2y - 3xy^2) = N$$

$$\frac{\partial N}{\partial x} = (12xy - 3y^2)$$

or
 $\nabla \times F = 0$



on C_1 : $y=2$
 $dy=0$ $1 \leq x \leq 3$

$$I_1 = \int_1^3 (24x - 4) dx = 12x^2 - 4x \Big|_1^3$$

$$= 108 - 12 - 12 - 4 = 80$$

on C_2 : $x=3$ $2 \leq y \leq 4$
 $dx=0$

$$I_2 = \int_2^4 (18y - 9y^2) dy = 9y^2 - 3y^3 \Big|_2^4$$

$$96 - 12 = 84$$

$$I = I_1 + I_2 = 164$$



$$(8) \quad I = \int_{(1,0)}^{(2,1)} (2xy - y^4 + 3) dx + (x^2 - 4xy^3) dy$$

$$\frac{\partial M}{\partial y} = 2x - 4y^3$$

$$\frac{\partial N}{\partial x} = 2x - 4y^3$$

$$\vec{F} = \vec{\nabla} \phi$$

$$\phi_x = 2xy - y^4 + 3 \rightarrow \phi = x^2y - xy^4 + 3x + A(y) \quad \text{Zero} \rightarrow$$

$$\phi_y = x^2 - 4xy^3 \rightarrow \phi = x^2y + y^4 + \underbrace{A_2(x)}_{= 3x}$$

$$\phi = x^2y - xy^4 + 3x$$

$$I = \phi(2,1) - \phi(1,0) = (4 - 2 + 6) - (0 - 0 + 3)$$

$$= +5$$